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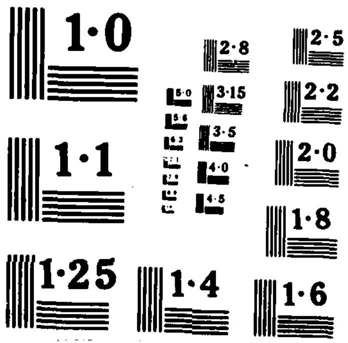
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THE UNIVERSITY OF ILLINOIS AT CHICAGO

A FAMILY OF LOCALLY RESISTANT NONSYMMETRIC
BIB DESIGNS OF DEGREE k

BY

A. HEDAYAT AND H. OHMORI

RESEARCH REPORT IN STATISTICS

No. 86-01

February 1986

DEPARTMENT OF MATHEMATICS, STATISTICS, AND COMPUTER SCIENCE

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The existence of a symmetric BIB(4t+3,2t+1,t) design implies the existence of a locally resistant BIB(4t+4,8t+6,4t+3,2t+2,2t+1) design of degree 2t+2. There are infinite number of such designs. Such designs are useful for running experiment under hostile cicumstances where there are good chance of losing one or more observations. Such designs will preserve the statistical optimality of the data . Other theoretical results are also given.

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A FAMILY OF LOCALLY RESISTANT NONSYMMETRIC
BIB DESIGNS OF DEGREE k

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Abstract

The existence of a symmetric BIB $(4t+3, 2t+1, t)$ design implies the existence of a locally resistant BIB $(4t+4, 8t+6, 4t+3, 2t+2, 2t+1)$ design of degree $2t+2$. There are infinite number of such designs. Other results are also given.

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A FAMILY OF LOCALLY RESISTANT NONSYMMETRIC BIB DESIGNS OF DEGREE k

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1. Introduction and Background. Hedayat and John (1974) introduced and studied the concept of resistant and susceptible balanced incomplete block (BIB) designs based on v treatments in b blocks of size k each. For convenience, we shall recall two of their concepts. They defined a BIB design to be locally resistant of degree $n \leq v-2$ if upon deletion of the experimental units assigned to a specific set of n treatments, the remaining design preserves the original property that under the usual homoscedastic additive linear model all normalized estimable linear functions of the remaining $v-2$ treatment effects are estimated with the same precision. Further, they defined a BIB design to be globally resistant of degree n if it has the above property with respect to all $\binom{v}{n}$ subsets of n treatments out of v treatments.

One obvious message which we can get from the results of Hedayat and John (1974) is this. Not all BIB designs, and certainly not all observations within a fixed BIB design are created equal. Specifically, their Theorem 5.5 states that a sufficient condition for a BIB design to be locally resistant of degree k is $v = b$. The necessity of this condition was

left as an open problem. In Section 2, we show that fortunately this condition is not necessary. We have identified a family of BIB designs with $v \neq b$ which are locally resistant of degree k. Therefore, our catalog of locally resistant BIB designs is substantially enlarged. In Section 3, we point out that some conditions stated in Hedayat and John (1974) can be improved upon and list some interesting and related papers which have appeared since 1974.

2. A Family of Locally Resistant BIB Designs of Degree k With $v \neq b$.

Theorem 2.1. The existence of a symmetric BIB (4t+3, 2t+1, 1) design implies the existence of a locally resistant BIB (4t+4, 8t+6, 4t+3, 2t+2, 2t+1) design of dgree 2t+2.

Proof. Let $\underline{N}$ and $\underline{M}$ be the incidence matrices of symmetrical BIB (4t+3, 2t+1, t) and BIB (4t+3, 2t+2, t+1) designs respectively. Note that the existence of $\underline{N}$ implies the existence of $\underline{M}$. Without loss of generality, $\underline{N}$ and $\underline{M}$ can be written as

$$\underline{N} = \begin{bmatrix} \underline{J}_{2t+1,1} & \underline{N}_1 \\ \underline{O}_{2t+2,1} & \underline{N}_2 \end{bmatrix}, \qquad \underline{M} = \begin{bmatrix} \underline{O}_{2t+1,1} & \underline{M}_1 \\ \underline{J}_{2t+2,1} & \underline{M}_2 \end{bmatrix}$$

where both $\underline{N}_2$ and $\underline{M}_2$ are incidence matrices of a

BIB (2t+2, 4t+2, 2t+1, t+1, t) design. Let

$$\bar{N} = \begin{bmatrix} 1 & J_{1,4t+2} & 0 & O_{1,4t+2} \\ J_{2t+1,1} & N_1 & O_{2t+1,1} & M_1 \\ O_{2t+2,1} & N_2 & J_{2t+2,1} & M_2 \end{bmatrix}.$$

Then $\bar{N}$ is the incidence matrix of a BIB(4t+4, 8t+6, 4t+3, 2t+2, 2t+1) design (see Lemma 2.1 of Bhat and Shrikhande (1970)). If we lose all the experimental units related to the first 2t+2 treatments as specified by the first 2t+2 rows of $\bar{N}$ we will be left with a design whose incidence matrix is given by $[N_2\ J_{2t+2,1}\ M_2]$. The C matrix of this remaining design can be easily computed to be the following complete symmetric matrix

$$(4t+1)(I_{2t+2} - (2t+2)^{-1} J_{2t+2,2t+2}).$$

Therefore, by Lemma 1.3 of Hedayat and John (1974), $\bar{N}$ is the incidence matrix of a locally resistant BIB(4t+4, 8t+6, 4t+3, 2t+2, 2t+1) design of degree 2t+2.

It is known that there are infinitely many symmetric BIB(4t+3, 2t+1, t) designs. Therefore, there are infinitely many locally resistant nonsymmetric BIB design of degree k. Fortunately, the family of symmetric BIB design and the above family of nonsymmetric BIB designs are not the only locally resistant designs of degree k. For example, the BIB(6,10,5,3,2)

design

$$\begin{array}{cccccccccc} 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 \\ 2 & 2 & 3 & 4 & 4 & 3 & 4 & 5 & 4 & 5 \\ 3 & 5 & 6 & 5 & 6 & 4 & 6 & 6 & 5 & 6 \end{array}$$

which is not a member of the family of designs in Theorem 2.1 is locally resistant of degree $k = 3$ with respect to any set of treatments within the same block.

An obvious way of generalizing the outcome of the above example is this. Suppose a BIB design, d, has the property that its b blocks can be partitioned into 3 sets of blocks, say S_1, S_2 and S_3 such that: (i) All blocks in S_1 are identical (S_1 can have a single block); (ii) Each block in S_2 has $k-1$ treatments in common with a block in S_1; (iii) If we remove those treatments which appear in blocks of S_1 from the blocks of S_3 then the remaining part of S_3 forms a BIB design with or without repeated blocks. Then d is a locally resistant design of degree k with respect to those k treatments which appear in S_1.

3. Closing Remarks

We like to point out that Corollary 4.2 of Hedayat and John (1974) could be improved upon by adding $(N_4)\ b \geq v + r - 1$ and replacing $(N_3)\ \lambda > 1$ by $(N_3)\ \lambda \geq k-1$.

Since 1974 several papers have appeared on this topic. We would like to mention the following selected papers. John (1976)

analyzed the performance of BIB designs upon the loss of an arbitrary treatment and proved that, in general, BIB designs are highly efficient in the sense of A-optimality among all equireplicated designs. Most (1975) generalized Theorem 4.3 of Hedayat and John (1974) characterizing globally resistant designs of degree n, $n < k < v$. Constantine (1986) proved that a BIB design preserves its optimality by loss of up to t-2 treatments if and only if it is a t-design. For a survey article on t-designs see Hedayat and Kageyama (1980) and Kageyama and Hedayat (1983).

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